Oscilatori

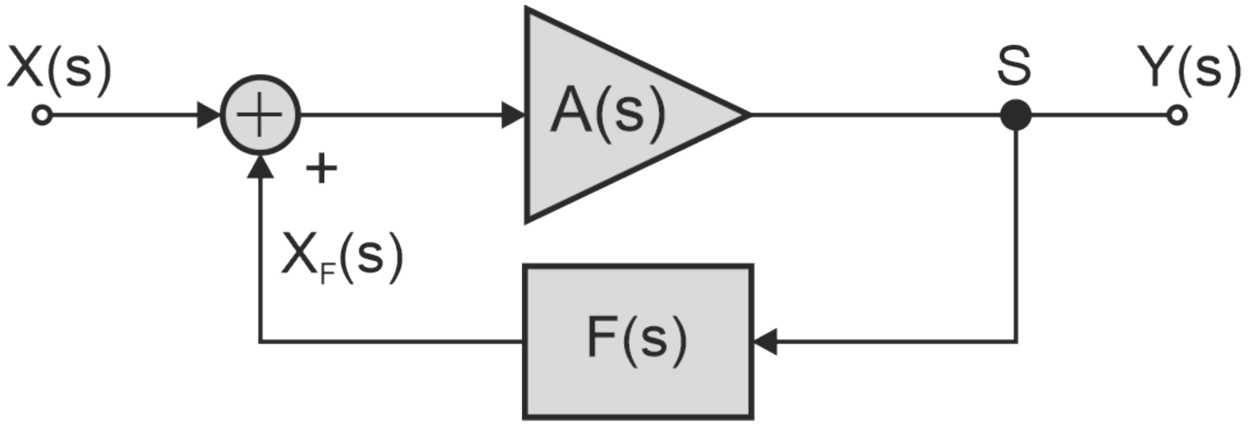
Uvod

- Oscilatori su kola koja (bez pobude) generišu periodičan signal (struju, napon) konstantne frekvencije.
- Generisan signal može biti prostoperiodičan ili složenoperiodičan.
- Oscilatori prostoperiodičnih signala se nazivaju *linearni oscilatori*. Oscilatori koji generišu složenoperiodične signale se nazivaju *relaksacioni oscilatori*.
- Osiclatori su kola sa *pozitivnom povratnom spregom*.
- Prema rezonantnom kolu, dele se na RC oscilatore, LC oscilatore i oscilatore sa kristalom kvarca

Uvod

- Pojačanje $A(s)$ i prenosna funkcija kola povratne sprege $F(s)$ su u opštem slučaju funkcije frekvencije s.

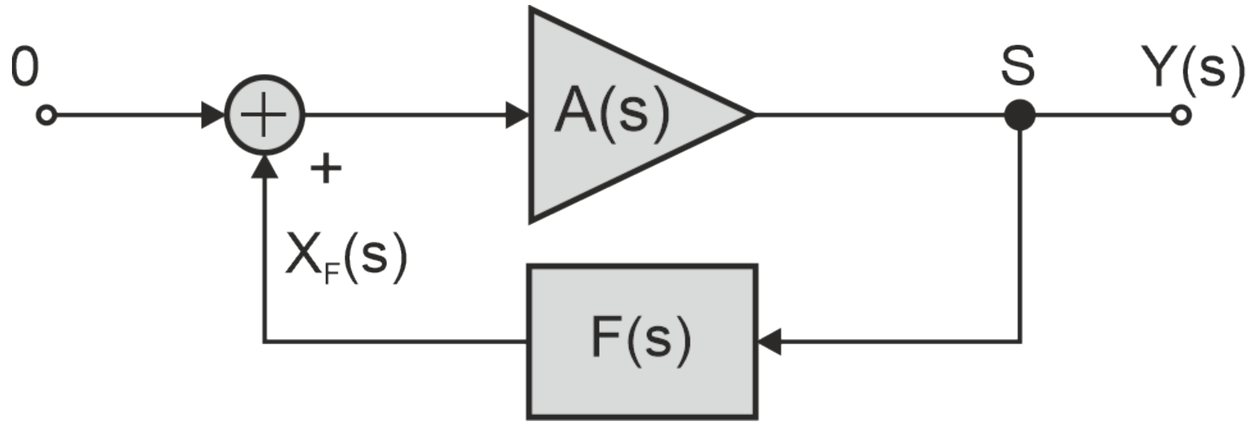
$$A_F(s) = \frac{A(s)}{1 - A(s)F(s)}$$



Uslov oscilovanja

- Ukoliko je kružno pojačanje $A(s)F(s)$ jednako jedinici, ukupno pojačanje je beskonačno.
- Kolo će bez pobude generisati signal $Y(s)$.
- *Barkhauzenov uslov* oscilovanja:

$$A(s)F(s)=1$$

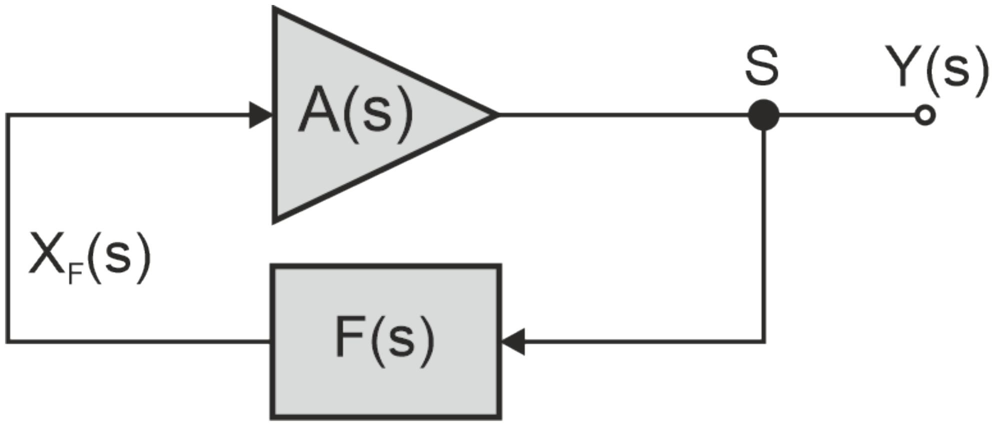


Barkhauzenov kriterijum

- Kružno pojačanje $A(s)F(s)$ je kompleksna funkcija, tako da se Barkhauzenov uslov može predstaviti realnim jednačinama:

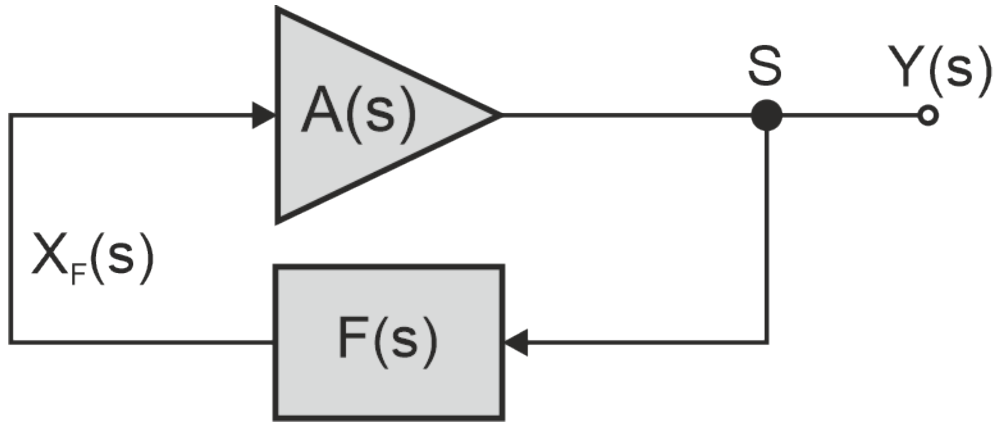
$$\operatorname{Re}\big(A(s)F(s)\big)=1,\quad \operatorname{Im}\big(A(s)F(s)\big)=0$$

$$\big|A(s)F(s)\big|=1,\quad \arg\big(A(s)F(s)\big)=2k\pi$$



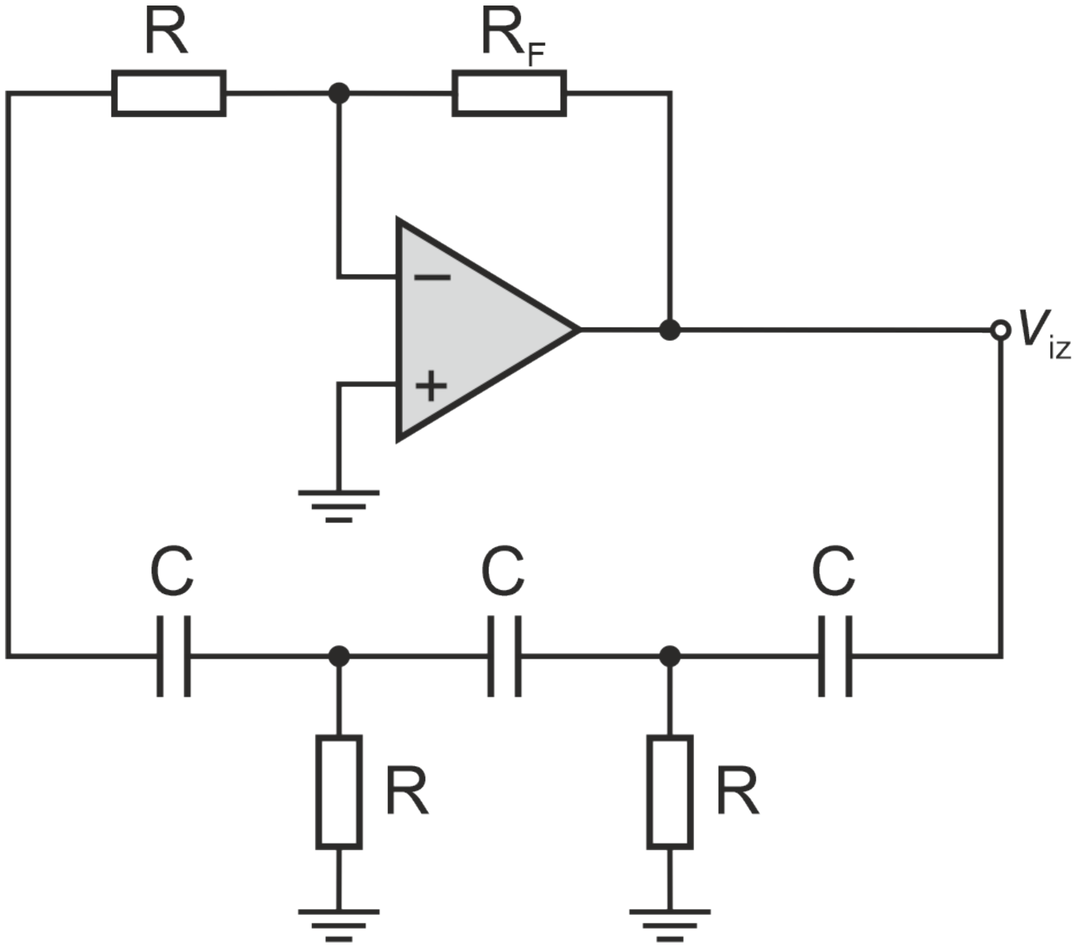
Analiza oscilatora

- Direktna primena Barkhauzenovog kriterijuma (određivanje kružnog pojačanja, uslova i frekvencije oscilovanja)
- Analiza stabilnosti kola, određivanjem matrice sistema jednačina kola



Oscilator sa faznim pomerajem

- Oscilator sa faznim pomerajem se sastoji od invertujućeg pojačavača i kola povratne sprege



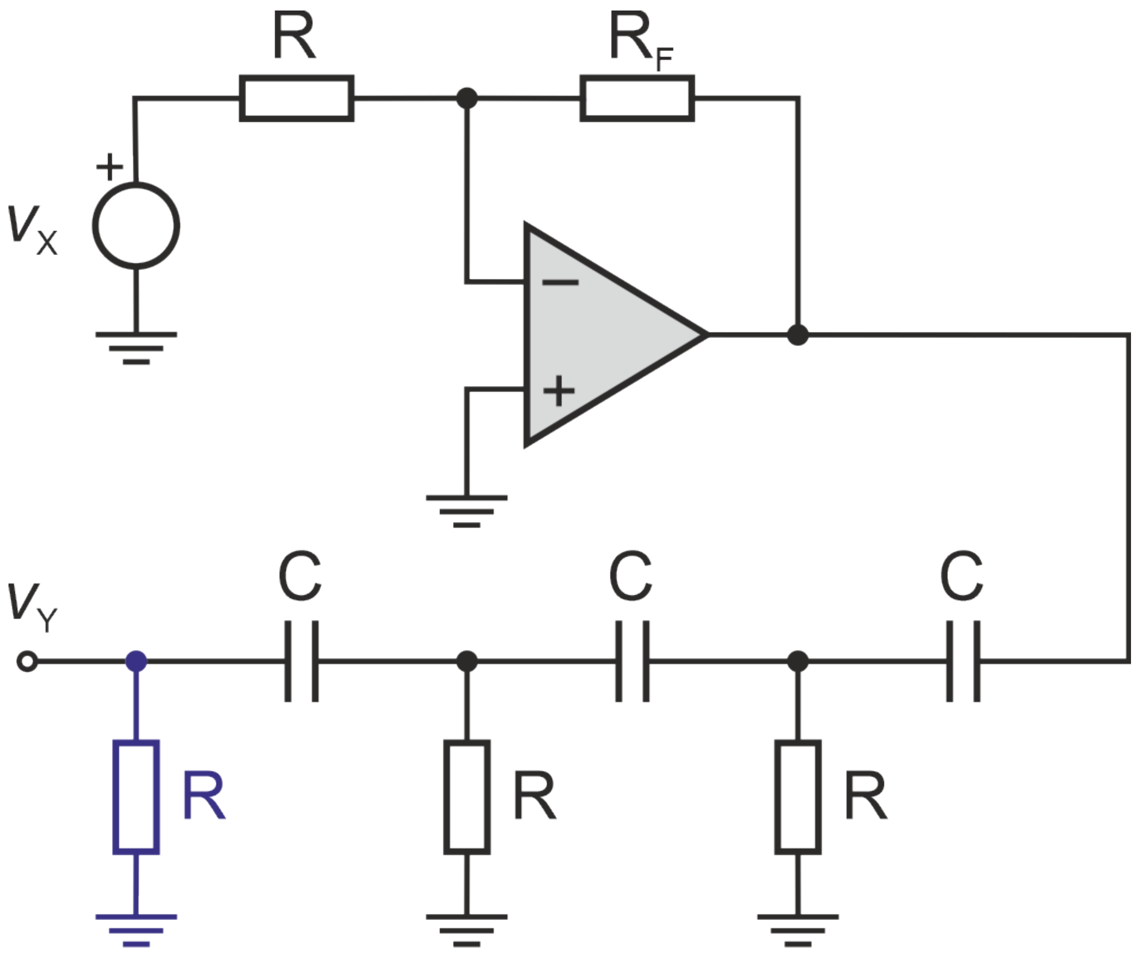
Oscilator sa faznim pomerajem

$$A(s)F(s) = \frac{v_Y(s)}{v_X(s)}$$

$$A(s) = -\frac{R_F}{R}$$

$$F(s) = \frac{R}{R + \frac{1}{sC}} \frac{Z_1}{Z_1 + \frac{1}{sC}} \frac{Z_2}{Z_2 + \frac{1}{sC}}$$

$$Z_1 = R \,||\, \left(R + \frac{1}{sC}\right), \quad Z_2 = R \,||\, \left(\frac{1}{sC} + Z_1\right)$$



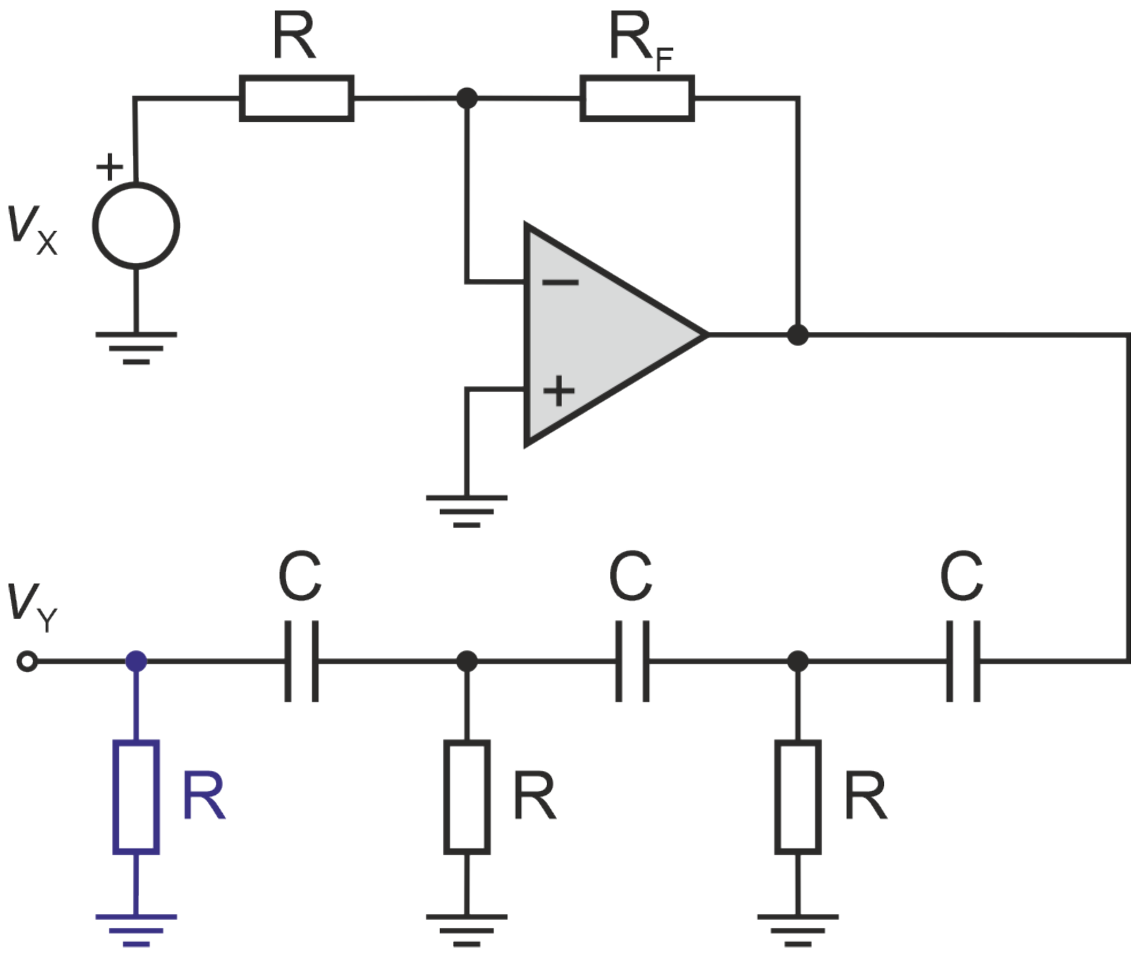
Oscilator sa faznim pomerajem

- Barkhauzenov uslov:

$$A(s)F(s)=1$$

$$A=-\frac{R_F}{R}$$

$$F(s)=\frac{s^3R^3C^3}{1+5sRC+6s^2R^2C^2+s^3R^3C^3}$$



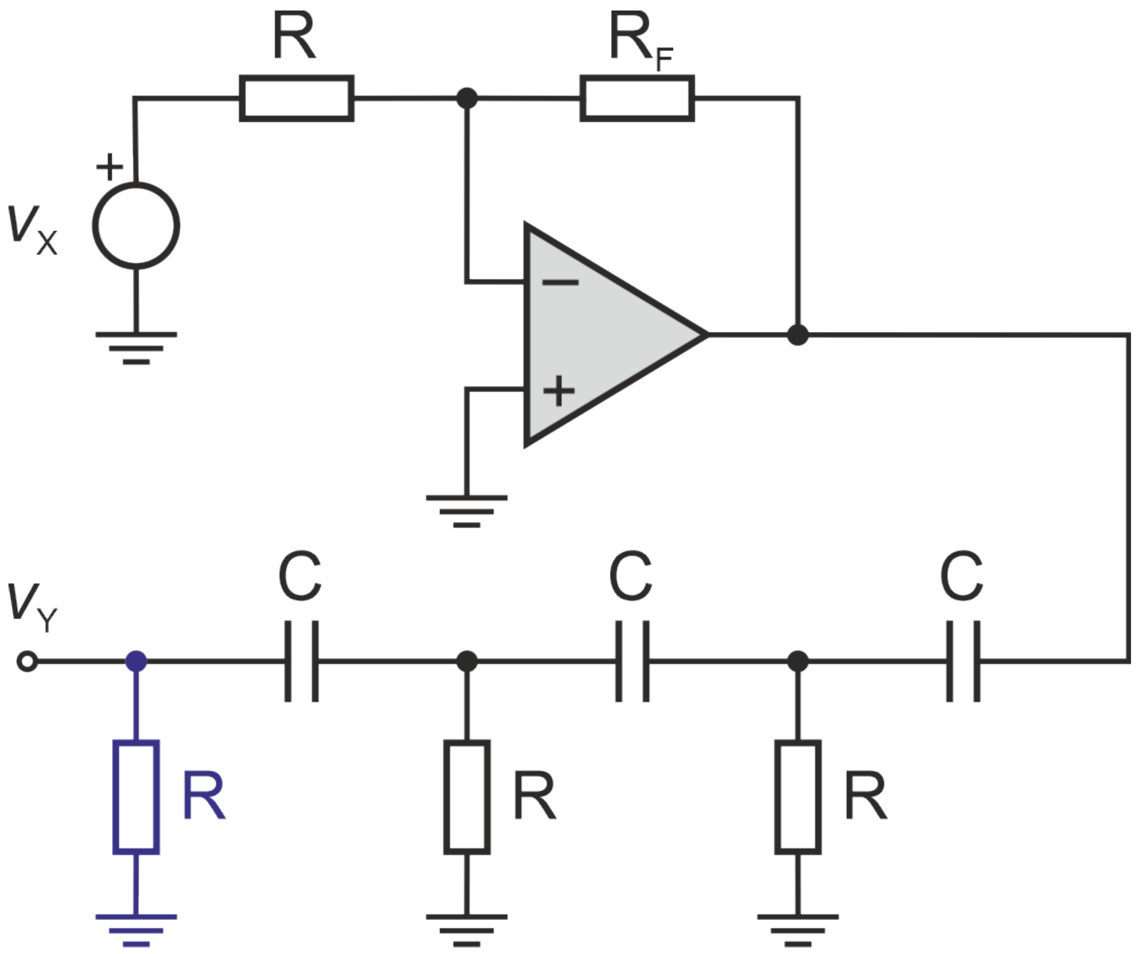
Oscilator sa faznim pomerajem

$$1 + 5sRC + 6s^2R^2C^2 + s^3R^3C^3 = As^3R^3C^3$$

$$s = j\omega, \quad \tau = RC$$

$$\mathrm{Re}: \quad 1 - 6\omega^2\tau^2 = 0$$

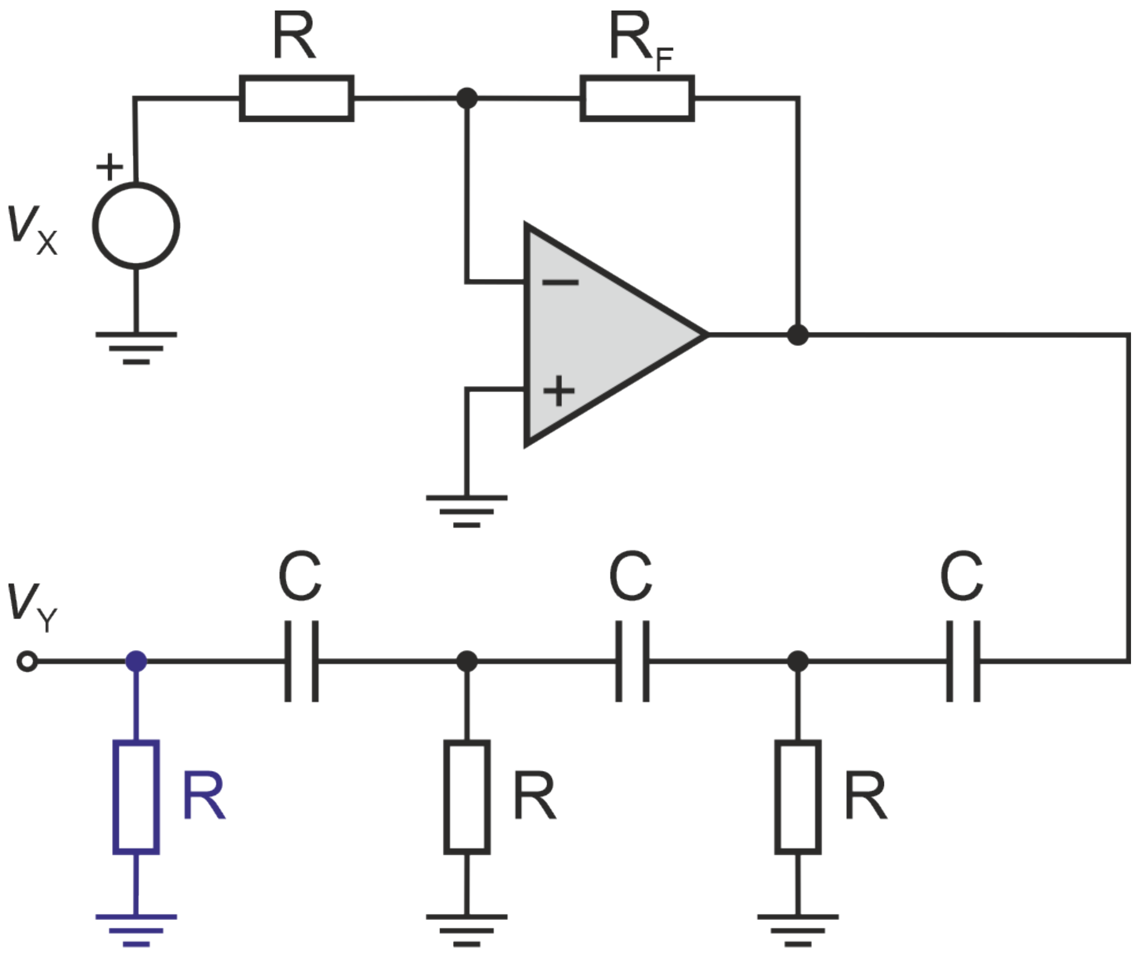
$$\mathrm{Im}: \quad 5\omega\tau - \omega^3\tau^3 = -A\omega^3\tau^3$$



Oscilator sa faznim pomerajem

Re: $\omega = \dfrac{1}{\sqrt{6}RC}$

Im: $A = -29, \quad R_F = 29R$

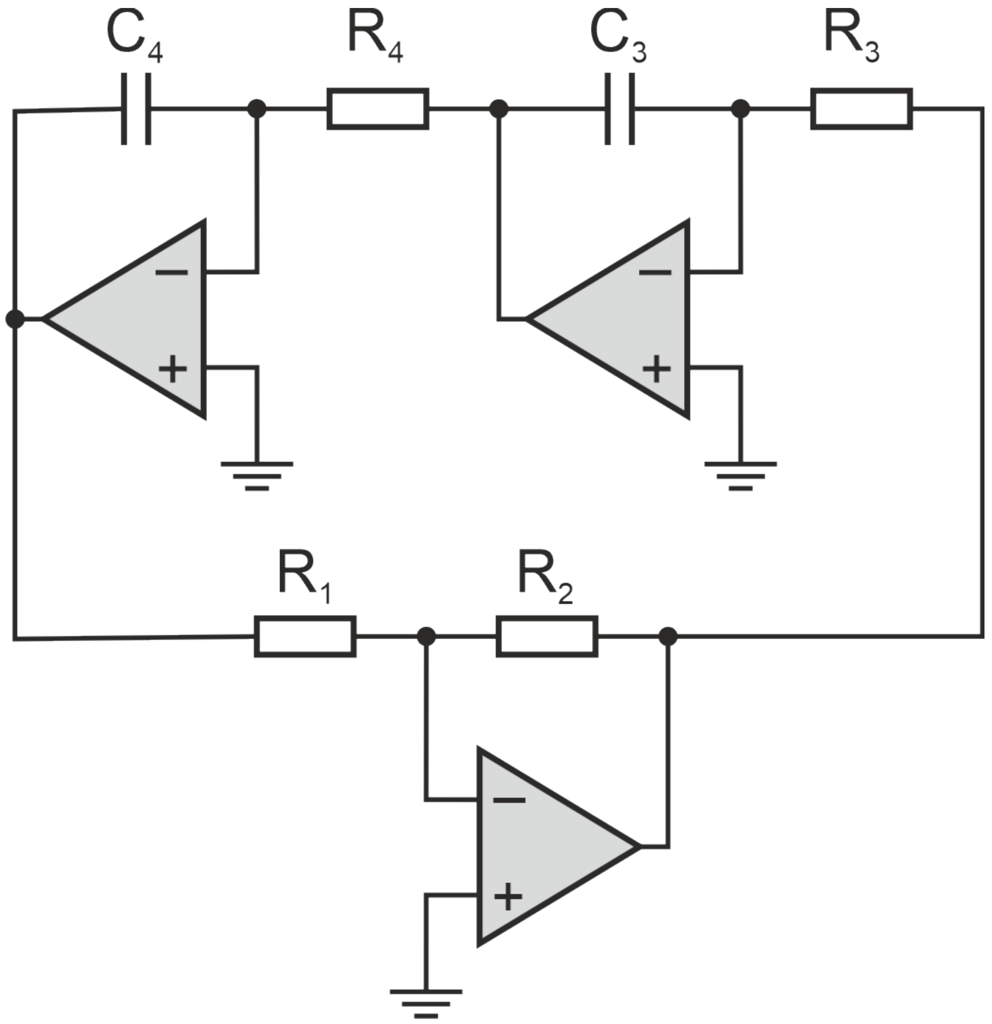


Oscilator sa dvostrukom integracijom

- Kolo povratne sprege je realizovano od dva kaskadno vezana integratora

$$A = -\frac{R_2}{R_1}$$

$$F(s) = \left(-\frac{1}{sR_3C_3}\right)\left(-\frac{1}{sR_4C_4}\right) = \frac{1}{s^2 R_3 R_4 C_3 C_4}$$



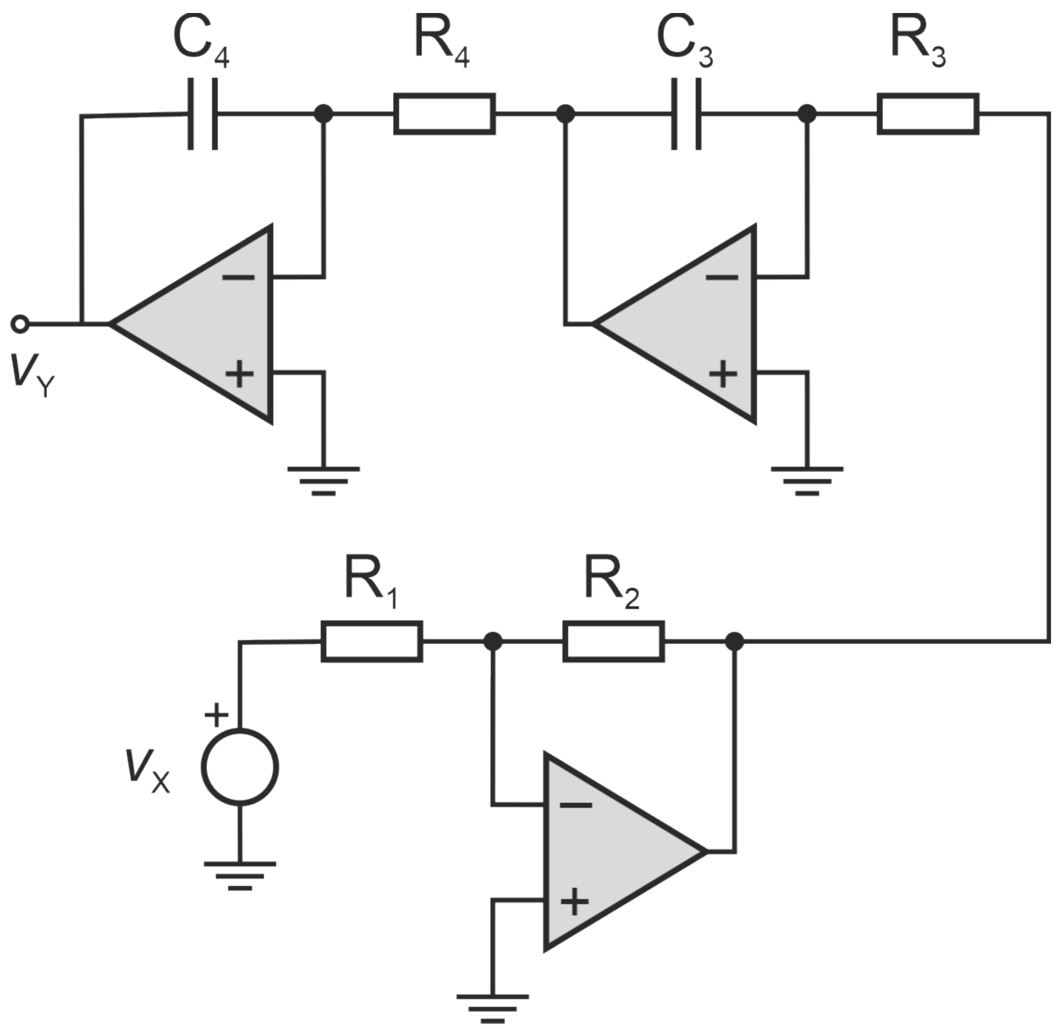
Oscilator sa dvostrukom integracijom

- Barkhauzenov kriterijum

$$\frac{v_Y}{v_X} = A(s)F(s) = 1, \quad s = j\omega$$

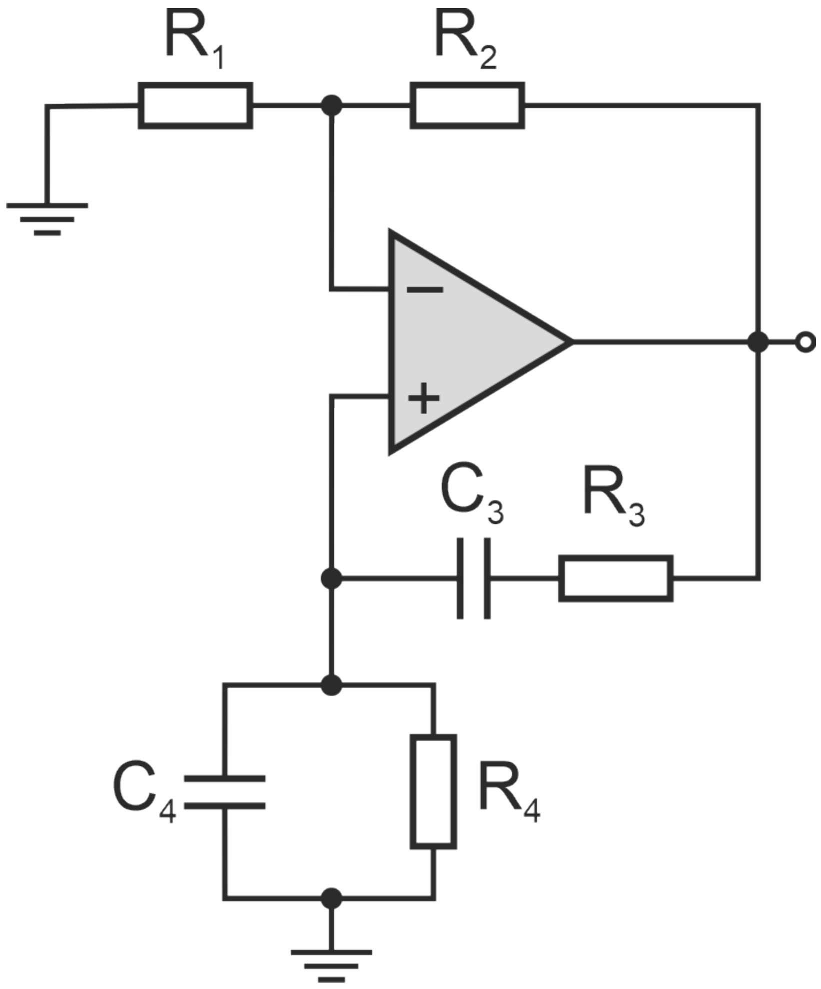
$$A(s)F(s) = -\frac{R_2}{R_1}\frac{1}{s^2 R_3 R_4 C_3 C_4}$$

$$\omega = \sqrt{\frac{R_2}{R_1 R_3 R_4 C_3 C_4}}$$



Oscilator sa Vinovim mostom

- Oscilator je realizovan primenom neinvertujućeg pojačavača, kolo povratne sprege je razdelnik napona paralelne (Z_4) i redne veze otpornika i kondenzatora (Z_3)



$$Z_3 = R_3 + \frac{1}{sC_3} = \frac{1 + sR_3C_3}{sC_3}$$

$$Z_4 = R_4 \,||\, \frac{1}{sC_4} = \frac{R_4}{1 + sR_4C_4}$$

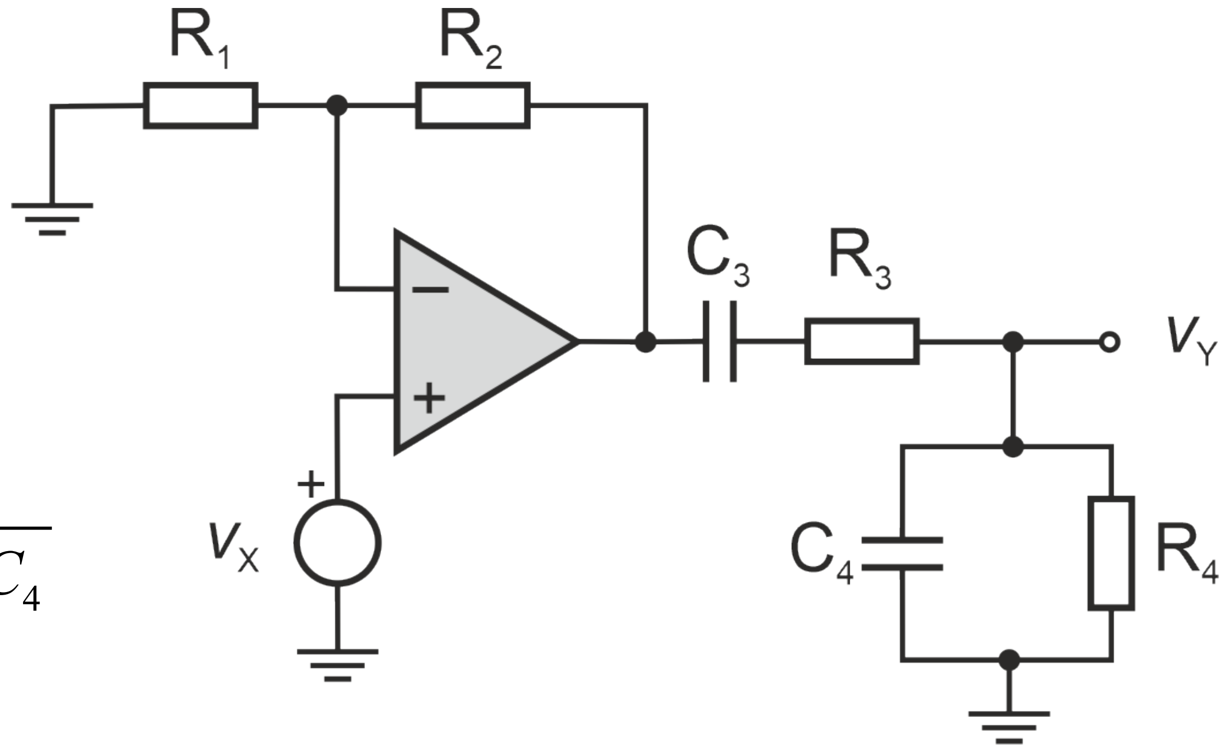
Oscilator sa Vinovim mostom

$$A = 1 + \frac{R_2}{R_1}$$

$$F(s) = \frac{Z_4}{Z_3 + Z_4}$$

$$F(s) = \frac{sR_4C_3}{1 + s\left(R_3C_3 + R_4C_3 + R_4C_4\right) + s^2 R_3R_4C_3C_4}$$

$$A(s)F(s) = \frac{v_Y}{v_X} = 1$$



Oscilator sa Vinovim mostom

$$sAR_4C_3 = 1 + s\left(R_3C_3 + R_4C_3 + R_4C_4\right) + s^2R_3R_4C_3C_4$$

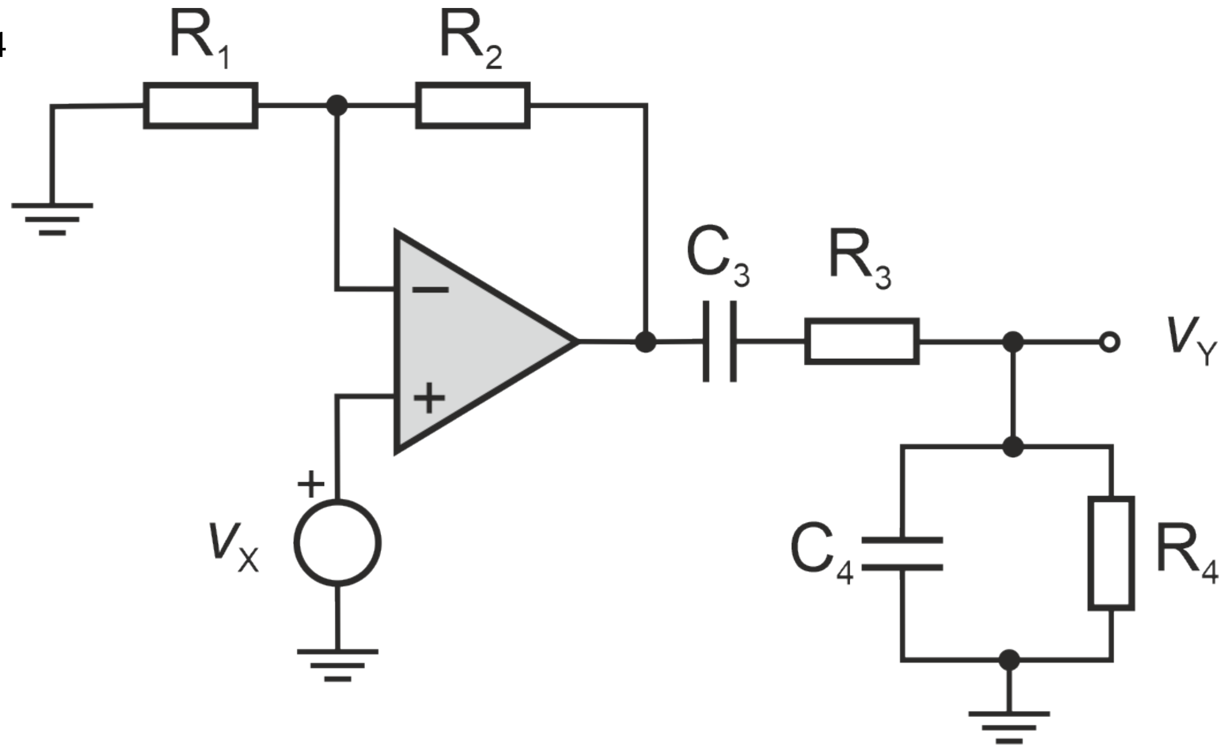
$$\mathrm{Re}: \quad 1 - \omega^2 R_3R_4C_3C_4 = 0$$

$$\boxed{\omega = \frac{1}{\sqrt{R_3R_4C_3C_4}}}$$

$$\mathrm{Im}: \quad AR_4C_3 = R_3C_3 + R_4C_3 + R_4C_4$$

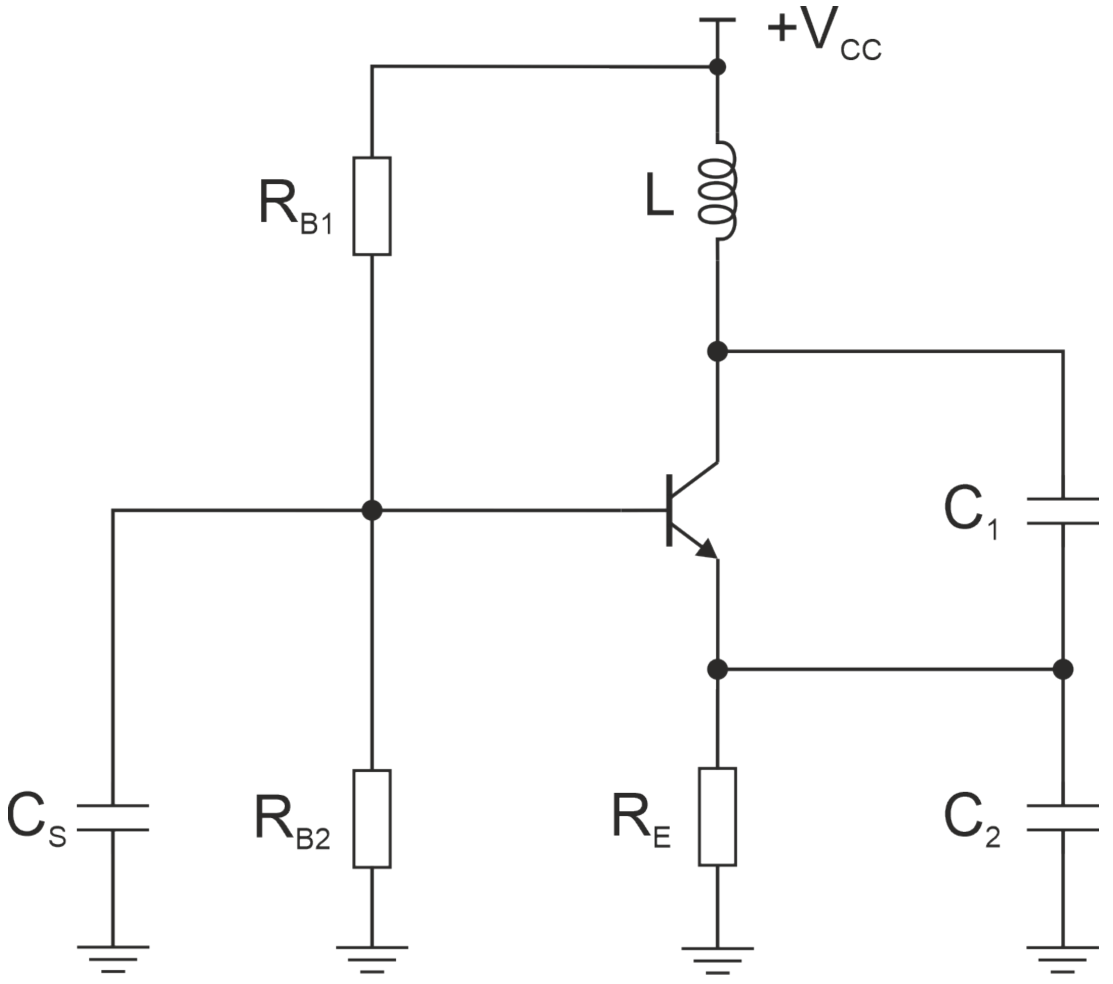
$$A = 1 + \frac{R_3C_3 + R_4C_4}{R_4C_3}$$

$$\boxed{\frac{R_2}{R_1} = \frac{R_3C_3 + R_4C_4}{R_4C_3} = \frac{R_3}{R_4} + \frac{C_4}{C_3}}$$



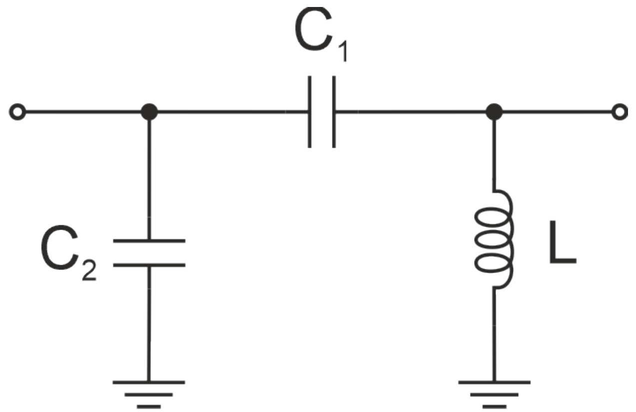
Kolpicov (Colpitts) oscilator

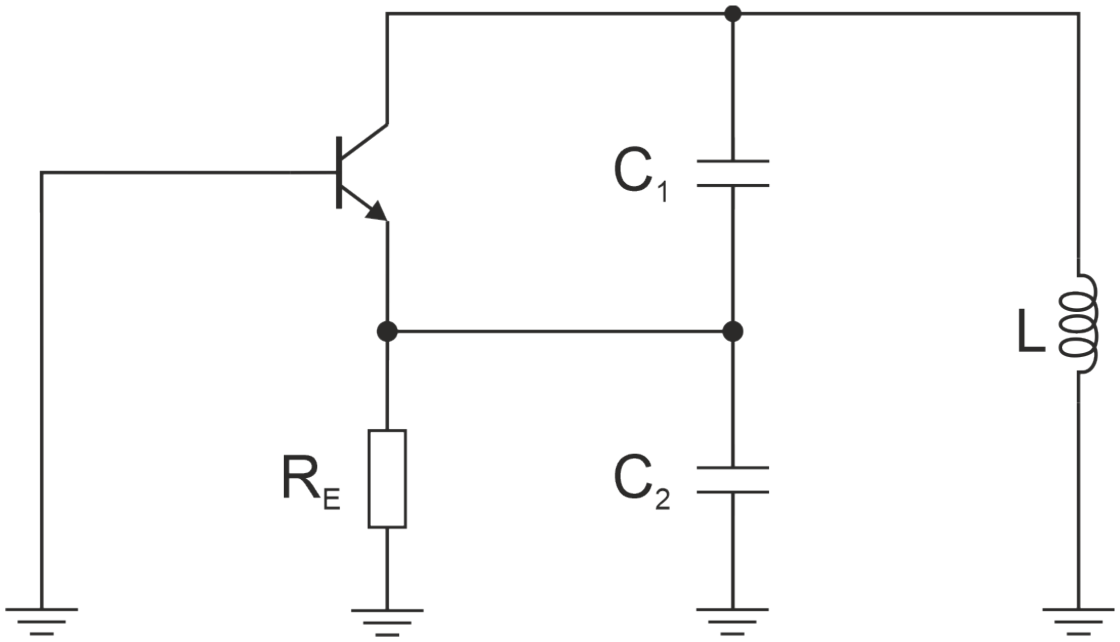
- LC oscilator
- Pojačavač sa zajedničkom bazom ili emitorm (gejtom ili sorsom)
- Ulaz je na emitoru, izlaz na kolektoru.
- Kolo povratne sprege čine kalem L i kondenzatori C_1 i C_2
- $C_S=\infty$, $r_o=\infty$
- Otpornici R_{B1} i R_{B2} služe za polarizaciju tranzistora



Kolpicov (Colpitts) oscilator

- Ekvivalentno kolo za naizmenični signal (desno)
- Kolo povratne sprege (dole)



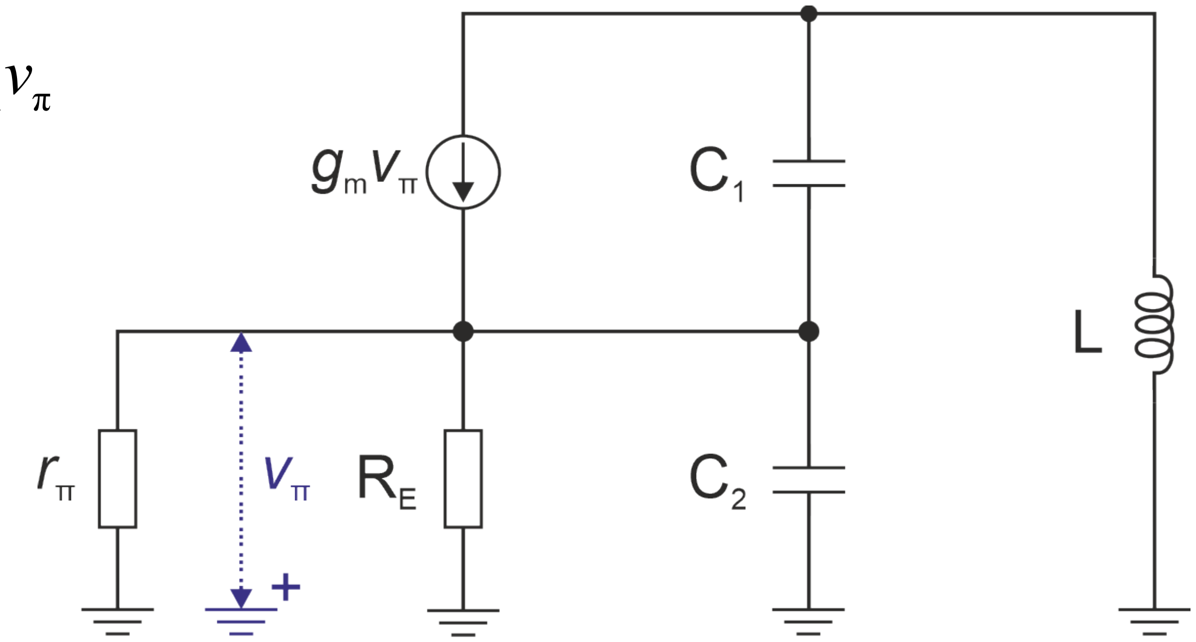


Kolpicov (Colpitts) oscilator

- Model bipolarnog tranzistora za male signale, $r_o=\infty$

E: $$-v_\pi\left(\frac{1}{r_\pi}+\frac{1}{R_E}+sC_1+sC_2\right)-v_C sC_1=g_m v_\pi$$

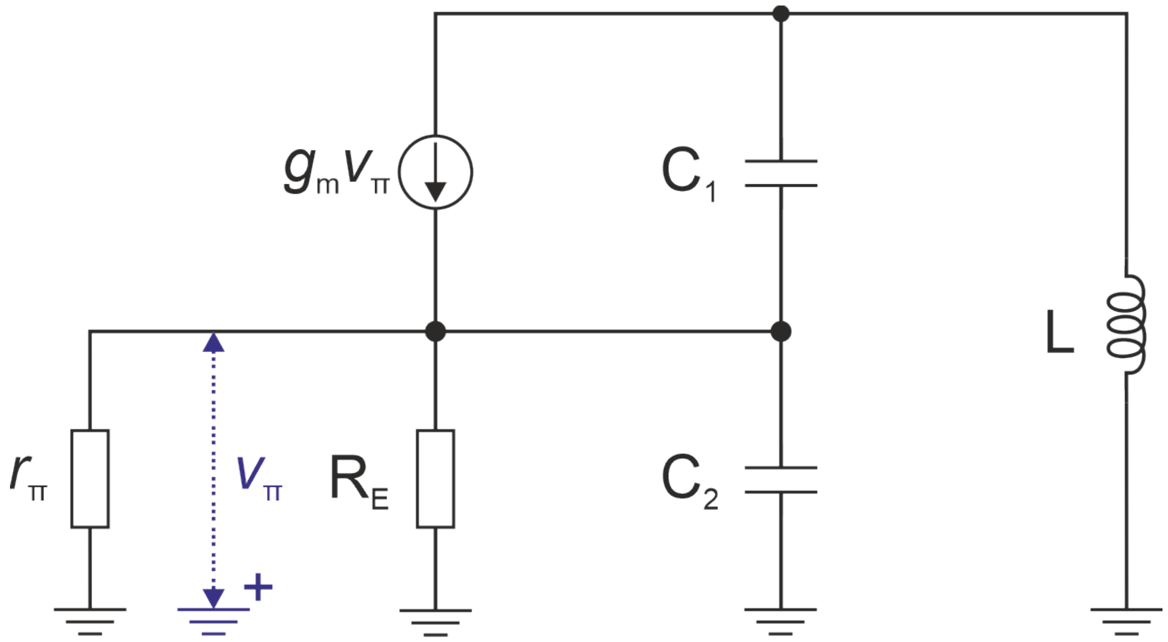
C: $$v_\pi sC_1+v_C\left(sC_1+\frac{1}{sL}\right)=-g_m v_\pi$$



Kolpicov (Colpitts) oscilator

E: $$v_{\pi}\left(\frac{1}{r_{\pi}}+\frac{1}{R_{E}}+sC_{1}+sC_{2}+g_{m}\right)+v_{C}sC_{1}=0$$

C: $$v_{\pi}\left(g_{m}+sC_{1}\right)+v_{C}\left(sC_{1}+\frac{1}{sL}\right)=0$$



Kolpicov (Colpitts) oscilator

- Slobodni vektor je nula vektor. Ukoliko je matrica sistema singularna, sistem ima beskonačno mnogo rešenja. Kolo je nestabilno.

$$\begin{bmatrix} 1/r_\pi + 1/R_E + sC_1 + sC_2 + g_m & sC_1 \\ g_m + sC_1 & sC_1 + 1/sL \end{bmatrix} \begin{bmatrix} v_\pi \\ v_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 1/r_\pi + 1/R_E + sC_1 + sC_2 + g_m & sC_1 \\ g_m + sC_1 & sC_1 + 1/sL \end{vmatrix} = 0$$

Kolpicov (Colpitts) oscilator

$$\begin{vmatrix} 1/r_\pi + 1/R_\mathrm{E} + sC_1 + sC_2 + g_\mathrm{m} & sC_1 \\ g_\mathrm{m} + sC_1 & sC_1 + 1/sL \end{vmatrix} = 0$$

$$s^2 C_1 C_2 + \left(sC_1 + \frac{1}{sL} \right)\left(g_\mathrm{m} + \frac{1}{r_\pi} + \frac{1}{R_\mathrm{E}} \right) - sC_1 g_\mathrm{m} + \frac{C_1 + C_2}{L} = 0$$

$$\omega = \sqrt{\frac{1}{L}\left(\frac{C_1 + C_2}{C_1 C_2} \right)}$$

$$\mathrm{Re:} \quad s^2 C_1 C_2 + \frac{C_1 + C_2}{L} = 0$$

$$-\omega^2 C_1 C_2 + \frac{C_1 + C_2}{L} = 0$$

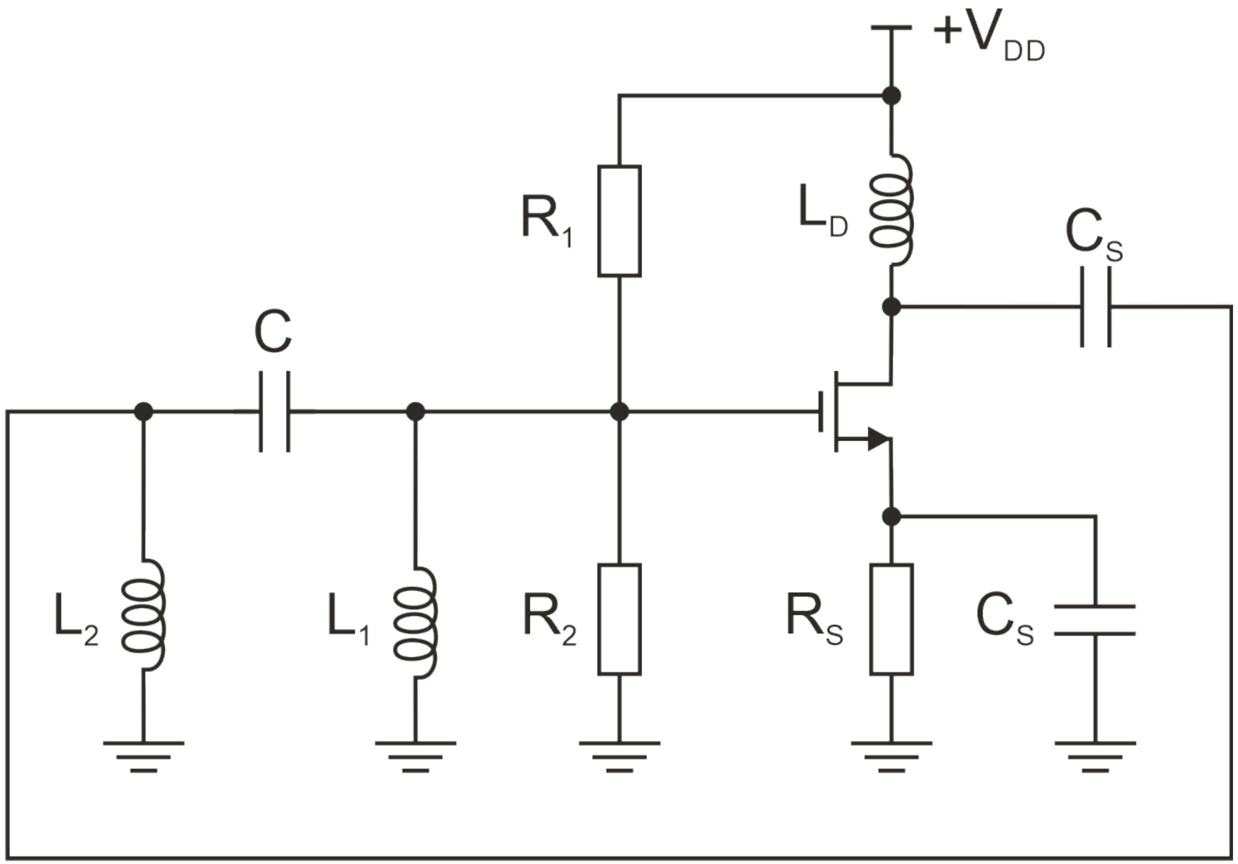
Kolpicov (Colpitts) oscilator

$$\text{Im}: \quad \left(sC_1 + \frac{1}{sL}\right)\underbrace{\left(g_\text{m} + \frac{1}{r_\pi} + \frac{1}{R_\text{E}}\right)}_{G_\text{X}} - sC_1 g_\text{m} = 0$$

$$\boxed{\frac{C_1}{C_2} = g_\text{m}\left(r_\pi \parallel R_\text{E}\right)}$$

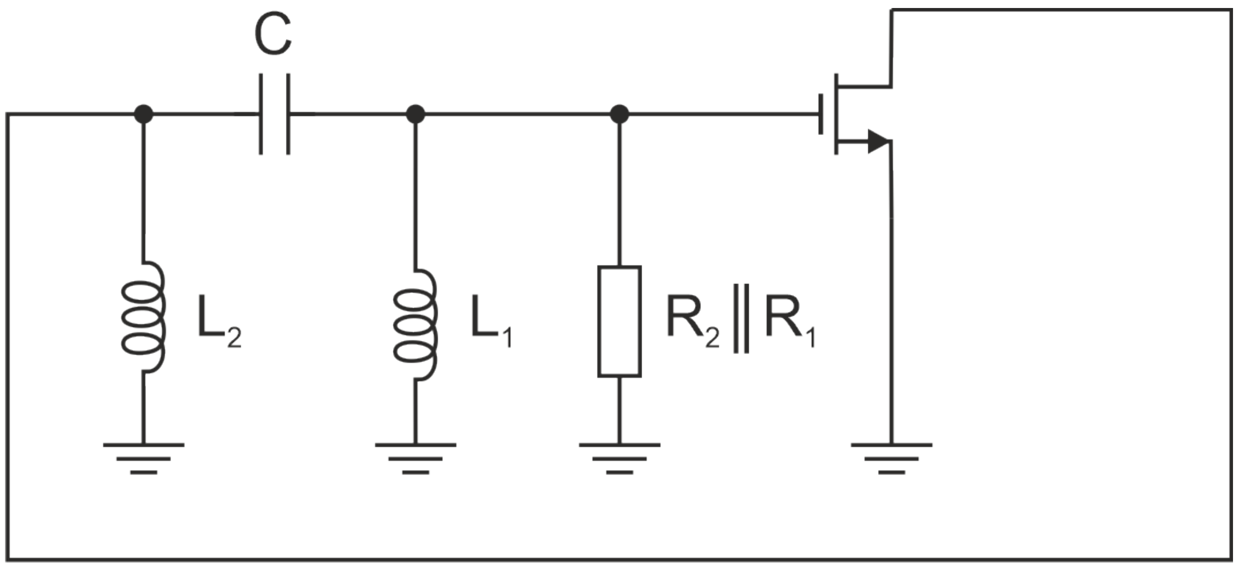
Hartlijev (Hartley) oscilator

- LC oscilator
- Pojačavač sa zajedničkom bazom ili emitorm (gejtom ili sorsom)
- Kolo povratne sprege čine kondenzator C i kalemovi L_1 i L_2
- $C_S=\infty$, $L_D=\infty$
- Otpornici R_1 i R_2 služe za polarizaciju tranzistora



Hartlijev (Hartley) oscilator

- Ekvivalentno kolo za naizmenični signal



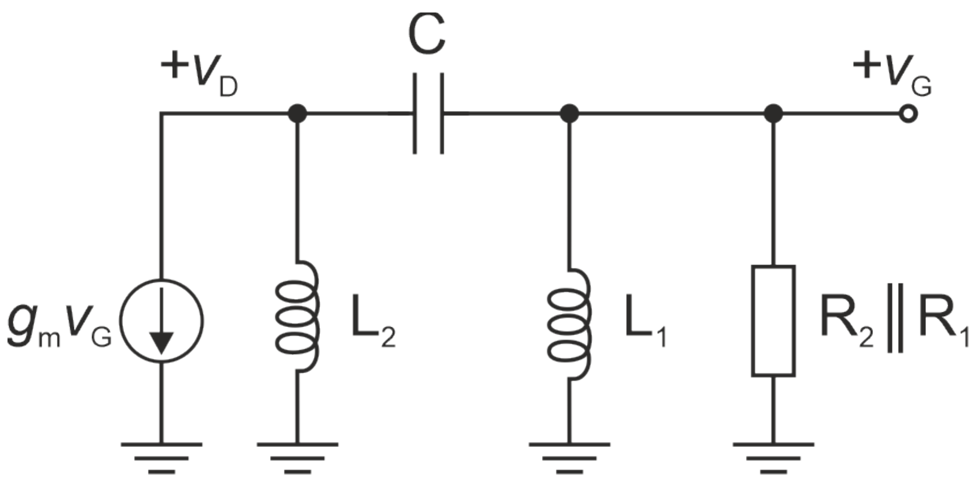
Hartlijev (Hartley) oscilator

- Model za male signale, $r_o=\infty$

$$R = R_1 \parallel R_2$$

D: $$v_D\left(\frac{1}{sL_2}+sC\right)-v_G sC=-g_m v_G$$

G: $$-v_D sC+v_G\left(\frac{1}{sL_1}+sC+\frac{1}{R}\right)=0$$



Hartlijev (Hartley) oscilator

D: $$v_D\left(\frac{1}{sL_2}+sC\right)+v_G\left(g_m-sC\right)=0$$

G: $$-v_D sC+v_G\left(\frac{1}{sL_1}+sC+\frac{1}{R}\right)=0$$

$$\begin{bmatrix}1/sL_2+sC & g_m-sC\\ -sC & 1/sL_1+1/R+sC\end{bmatrix}\begin{bmatrix}v_D\\ v_G\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

- Slobodni vektor je nula vektor. Ukoliko je matrica sistema singularna, sistem ima beskonačno mnogo rešenja. Kolo je nestabilno.

Hartlijev (Hartley) oscilator

$$\begin{vmatrix} 1/sL_2 + sC & g_m - sC \\ -sC & 1/sL_1 + 1/R + sC \end{vmatrix} = 0$$

$$\frac{1}{s^2 L_1 L_2} + \frac{1}{sRL_2} + \frac{C}{L_2} + \frac{C}{L_1} + \frac{sC}{R} + sCg_m = 0$$

$$\boxed{\omega = \frac{1}{\sqrt{C(L_1 + L_2)}}}$$

$$\text{Re}: \quad \frac{1}{s^2 L_1 L_2} + \frac{C}{L_2} + \frac{C}{L_1} = 0$$

$$-\frac{1}{\omega^2 L_1 L_2} + \frac{C}{L_2} + \frac{C}{L_1} = 0$$

Hartlijev (Hartley) oscilator

$$\frac{1}{s^2 L_1 L_2} + \frac{1}{sRL_2} + \frac{C}{L_2} + \frac{C}{L_1} + \frac{sC}{R} + sCg_{\mathrm{m}} = 0$$

$$\mathrm{Im}: \quad \frac{1}{sRL_2} + \frac{sC}{R} + sCg_{\mathrm{m}} = 0$$

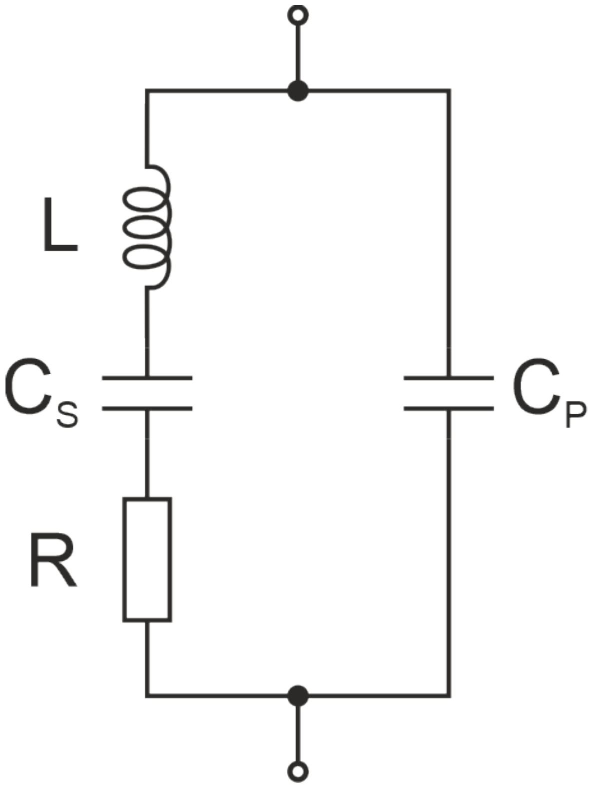
$$-\frac{1}{\omega RL_2} + \frac{\omega C}{R} + \omega Cg_{\mathrm{m}} = 0$$

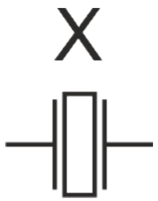
$$\boxed{g_{\mathrm{m}} R = \frac{L_1}{L_2}}$$

Oscilator sa kristalom kvarca

- Frekvencija oscilovanja LC i RC oscilatora zavisi od temperature, napona napajanja i starenja (tolerancije) komponenti.
- Određene primene zahtevaju preciznu i konstantnu frekvenciju, u tim slučajevima se koriste oscilatori sa kristalom kvarca.
- Kvarc je piezoelektrični materijal koji vibrira na određenoj frekvenciji.
- Frekvencija vibriranja je stabilna u odnosu na promenu temperature (≈1ppm/K).
- Kristal kvarca može biti precizno dimenzionisan, tako da je moguće postići željenu rezonantnu frekvenciju sa malom greškom (10ppm).
- Kristal kvarca ima male gubitke.

Model kristala kvarca





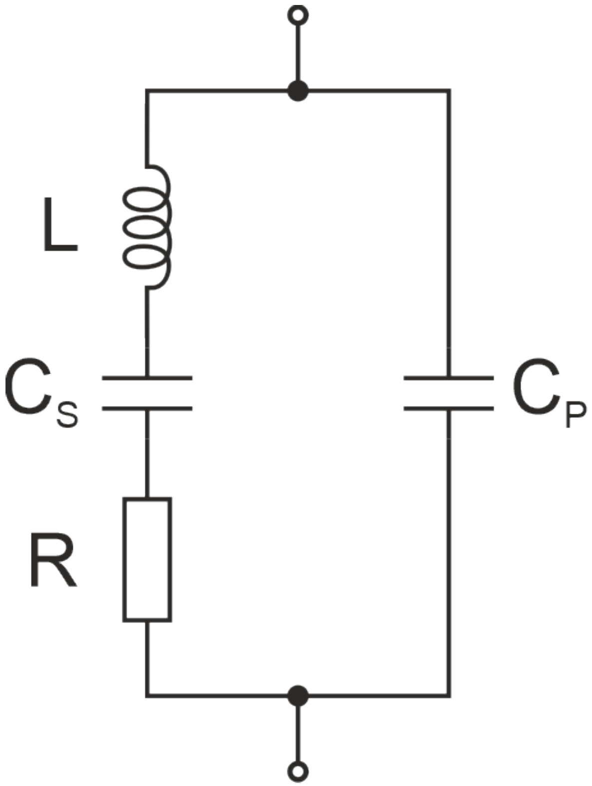
$$L = 1 \div 1000\,\mathrm{mH}$$

$$R = 10 \div 1000\,\Omega$$

$$C_P \approx 1\,\mathrm{pF}$$

$$C_S \approx 10^{-15} \div 10^{-13}\,\mathrm{F}$$

Model kristala kvarca

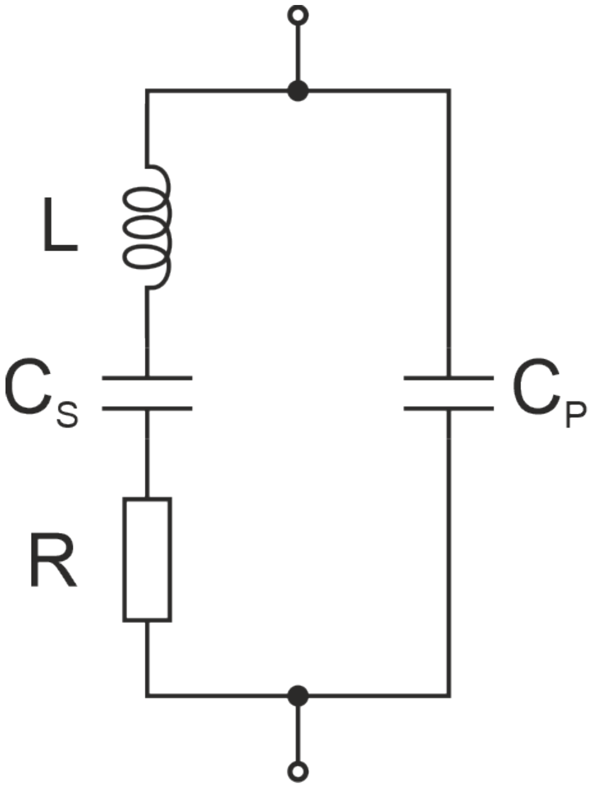


$$Z_X = \left(R + \frac{1}{sC_S} + sL \right) \| \frac{1}{sC_P}$$

$$Z_X = \frac{\frac{1}{sC_P}\left(R + \frac{1}{sC_S} + sL \right)}{R + \frac{1}{sC_S} + \frac{1}{sC_P} + sL}$$

$$Z_X = \frac{1 + sRC_S + s^2 LC_S}{s\left(C_S + C_P\right) + s^2 RC_S C_P + s^3 LC_S C_P}$$

Model kristala kvarca

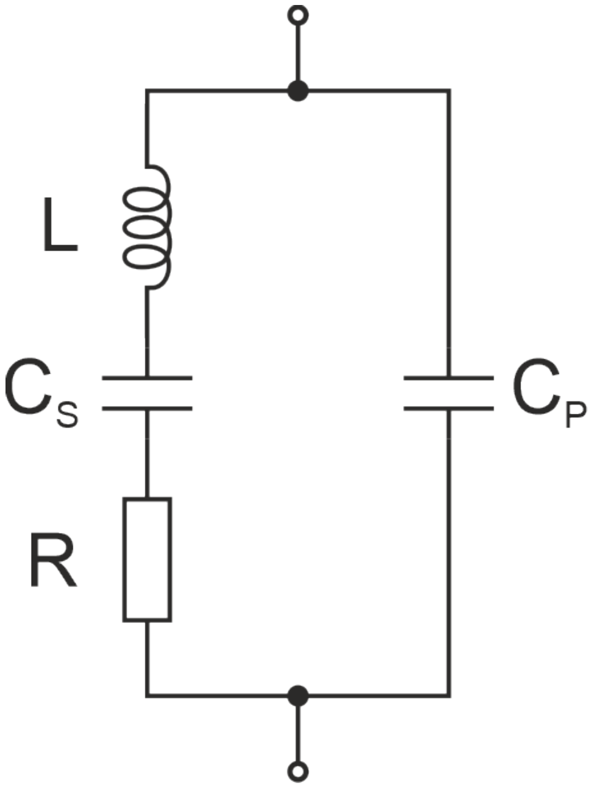


$$Z_X = \frac{1}{s\left(C_S + C_P\right)} \frac{1 + sRC_S + s^2 LC_S}{1 + sR\dfrac{C_S C_P}{C_S + C_P} + s^2 L \dfrac{C_S C_P}{C_S + C_P}}$$

$$Z_X = \frac{1}{s\left(C_S + C_P\right)} \frac{1 + sRC_S + s^2 LC_S}{1 + sRC_X + s^2 LC_X} \qquad C_X = \frac{C_S C_P}{C_S + C_P}$$

$$\omega_P = \frac{1}{\sqrt{LC_X}} \qquad \omega_S = \frac{1}{\sqrt{LC_S}}$$

Model kristala kvarca

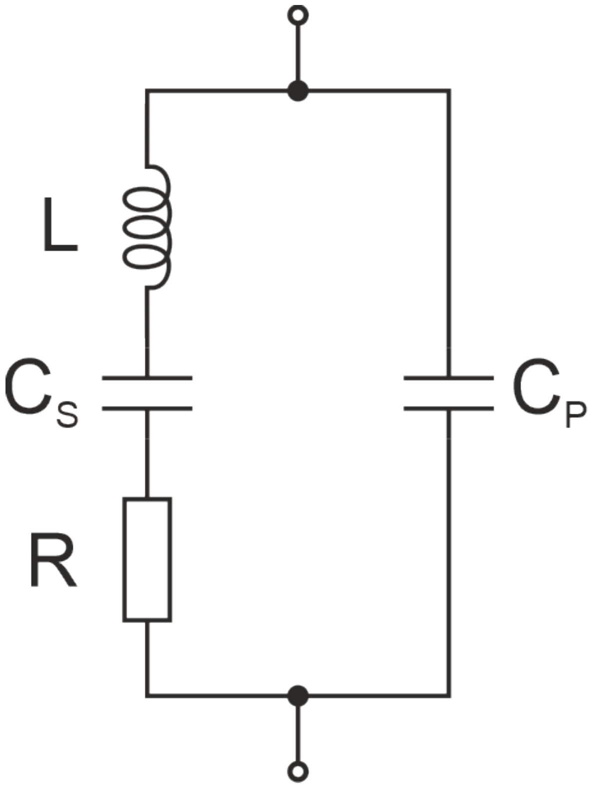


$$Z_X \approx \frac{1}{s\left(C_S + C_P\right)} \frac{1 + s^2 L C_S}{1 + s^2 L C_X}$$

$$\omega_P = \frac{1}{\sqrt{LC_X}} \qquad \omega_S = \frac{1}{\sqrt{LC_S}}$$

$$Z_X \approx \frac{1}{s\left(C_S + C_P\right)} \frac{1 - \left(\frac{\omega}{\omega_S}\right)^2}{1 - \left(\frac{\omega}{\omega_P}\right)^2}$$

Model kristala kvarca



$$Z_X \approx \frac{1}{s\left(C_S + C_P\right)} \frac{1 + s^2 L C_S}{1 + s^2 L C_X}$$

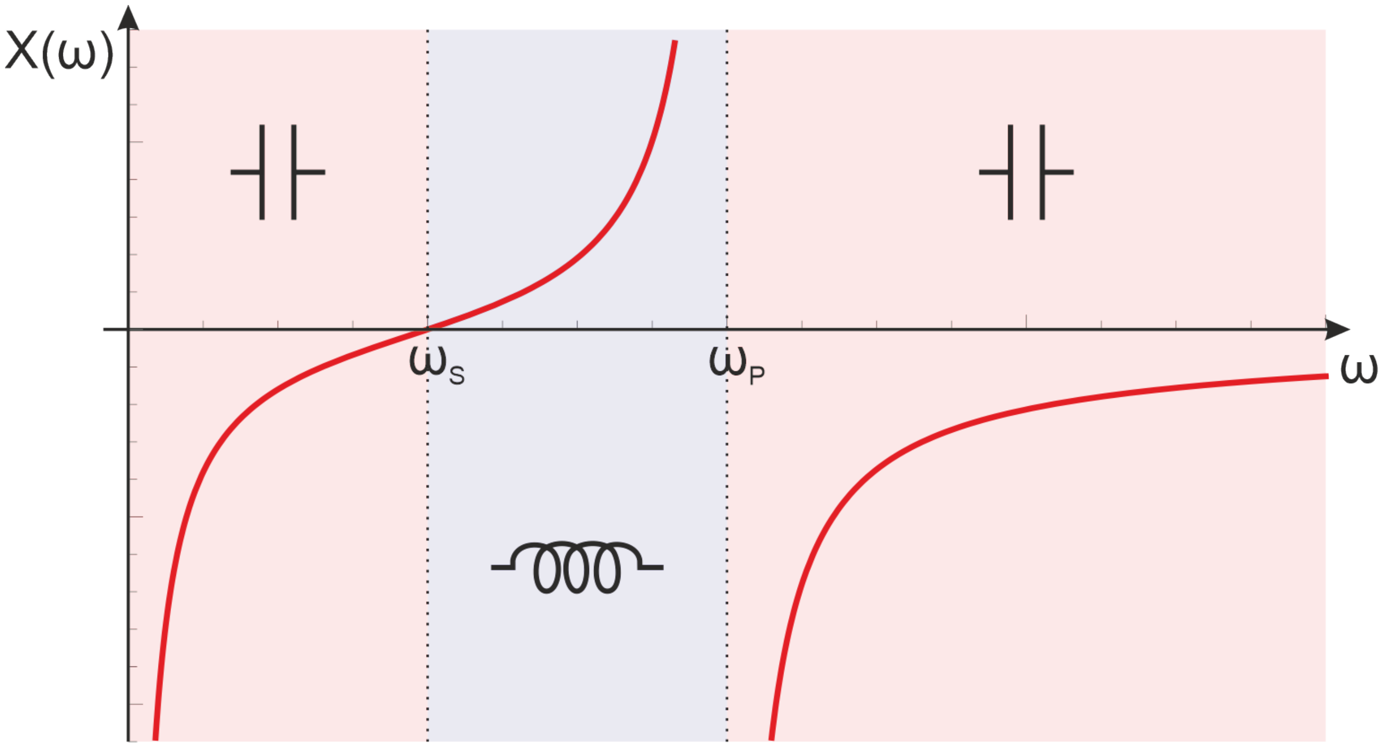
$$\omega_P = \frac{1}{\sqrt{L C_X}} \qquad \omega_S = \frac{1}{\sqrt{L C_S}}$$

$$Z_X \approx \frac{1}{j\omega\left(C_S + C_P\right)} \frac{1 - \left(\frac{\omega}{\omega_S}\right)^2}{1 - \left(\frac{\omega}{\omega_P}\right)^2}$$

Model kristala kvarca

$$\omega_S = \frac{1}{\sqrt{LC_S}} \qquad \omega_P = \frac{1}{\sqrt{LC_X}}$$

$$Z_X = jX \approx \frac{-j}{\omega\left(C_S + C_P\right)} \frac{1-\left(\frac{\omega}{\omega_S}\right)^2}{1-\left(\frac{\omega}{\omega_P}\right)^2}$$



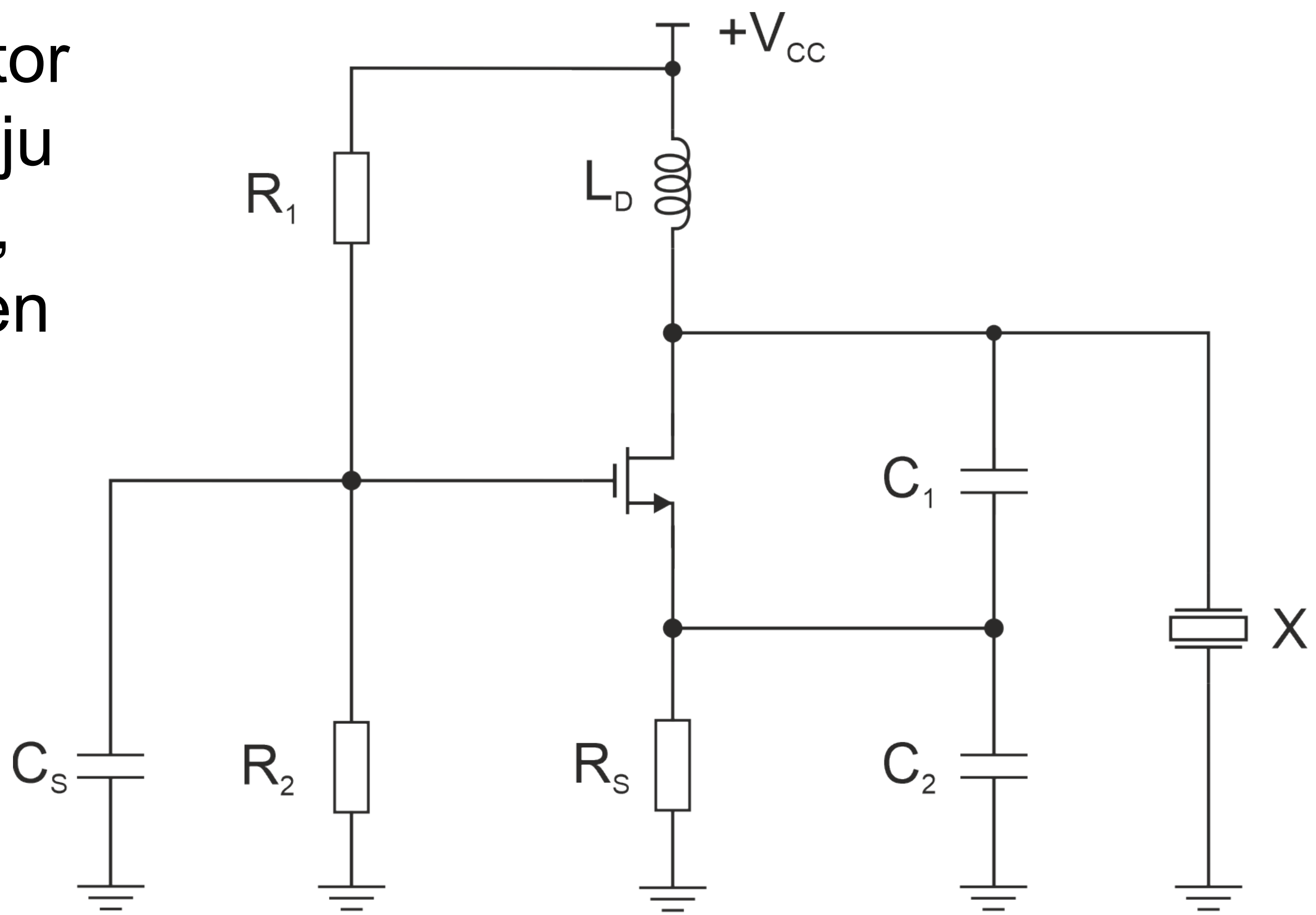
Pirsov (Pierce) oscilator

- Kristal kvarca ima induktivnu impedansu između rezonantnih frekvencija ω_s i ω_p.
- Razlika rezonantnih frekvencija ω_s i ω_p je mala, tako da se kristal ponaša kao kalem za mali opseg frekvencija. Unutar opsega induktivnost se značajno menja.
- U kolima oscilatora kristal kvarca zamenjuje kalem, tako da se frekvencija oscilovanja nalazi između rezonantnih frekvencija. Bira se oblast bliža rezonantnoj frekvenciji ω_p, gde promena frekvencije izaziva najveću promenu induktivnosti. Ova osobina kristala kvarca obezbeđuje stabilnost frekvencije oscilovanja.

Pirsov (Pierce) oscilator

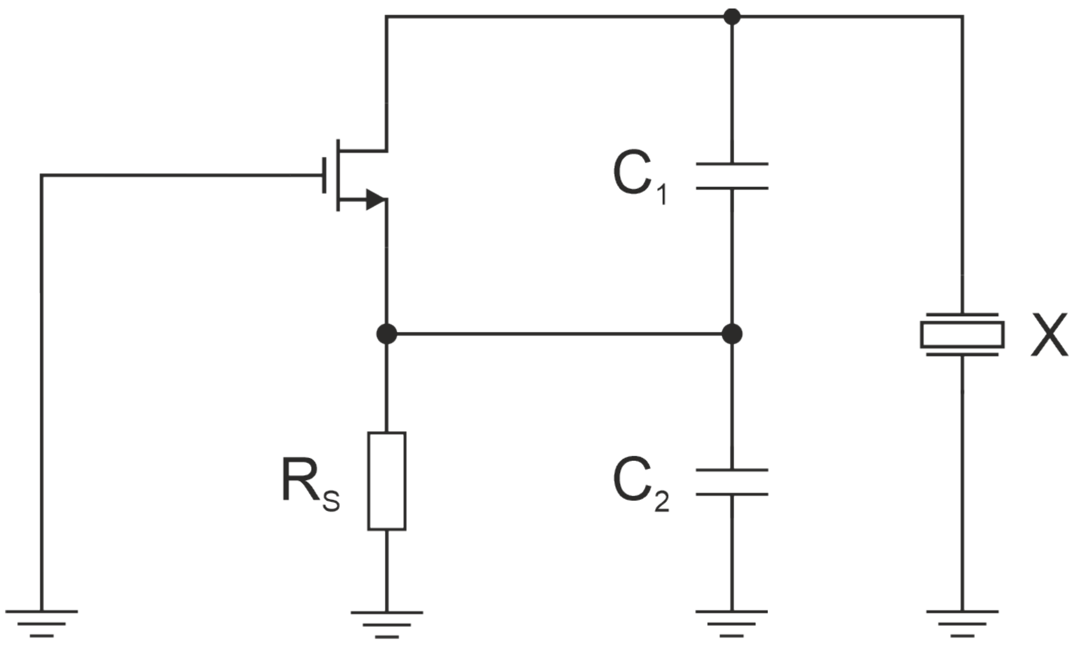
- Pirsov (Pierce) oscilator predstavlja modifikaciju Kolpicovog oscilatora, gde je kalem zamenjen kristalom kvarca

$$L_D=\infty, C_S=\infty$$



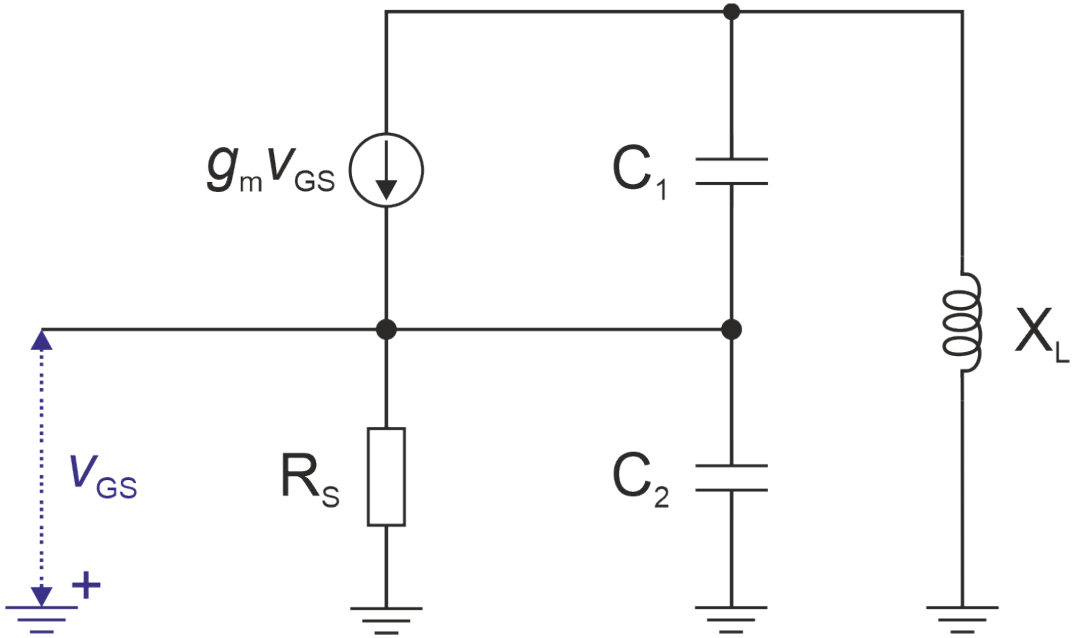
Pirsov (Pierce) oscilator

- Ekvivalentno kolo za naizmeničan signal



Pirsov (Pierce) oscilator

- Model za male signale ($r_o = \infty$)



Pirsov (Pierce) oscilator

- Pirsov oscilator sa CMOS pojačavačem